Lesson 4 Classical Monetary Model (1)

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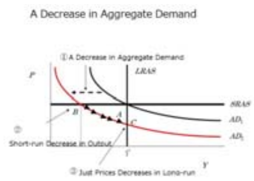
1 Price Stickiness



- Price stickiness explains how changes in nominal variables affect real variables.
- Price stickiness can be explained by ovserving micro data in changes in prices.

- Taylor (1999)
1. Average duration in price revision is approximately one year.
- Bils and Klenow (2004)
1. Duration in price revision in 350 product aggregates which compose the CPI is 4 to 6 months in average.
- Nakamura and Steinsson (2006)
1. Review Bils and Klenow (2004) by excluding sale products.
2. Average duration in price revision is 8 to 11 months.

- Dhyne et al. (2006)
1. Data in Euro zone
2. The duration is different among product aggregates.
3. Service prices' duration is long while perishable foods and energy prices' duration is short.

2 Monetary Policy Neutrality

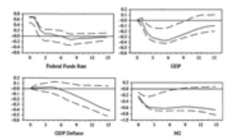
- If followings are applied, the CB can control aggregate demand through monetary policy.
1. Changes in money supply do not correspond to changes in prices one by one.

$$\frac{M_t}{P_t} = L\left(\overset{(-)}{i_t}, \overset{(+)}{Y_t} \right)$$

2. Changes in nominal interest rate do not correspond to expected inflation one by one.

$$i_t = r_t + \mathrm{E}_t\left(\pi_{t+1}\right) \qquad \beta\left(\frac{u'(C_{t+1})}{u'(C_t)}\right) = \frac{1}{1+r_t}$$

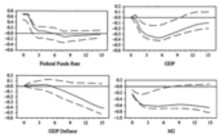
Fig. 4-1: Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

- Christiano, Eichenbaum and Evans (1999) estimate how exogenous monetary policy shock affects macroeconomic variables.
- Here, monetary policy shock is a residual between FF rate and its theoretical value which is a function of current and past value in GDP, GDP deflator and commodity price index.

Fig. 4-1 : Estimated Dynamic Response to a Monetary Policy Shock

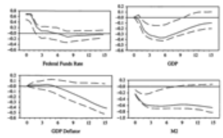


Source: Christiano, Eichenbaum and Evans (1999)

- To 75 basis point increase in FF rate (1 basis point is 0.01 percent point),

1. GDP decreases 50 basis points in 5 quarters (The nominal affects the real).
2. GDP deflator do not change over a year (Nominal rigiditiy).

Fig. 4-1 : Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

3. Money supply decreases (A decrease in demand for money through an increase in nominal interest rate).

- New Keynesian models introduce implications on data.

3 Classical Monetary Model

- Before lecturing New Keynesian model, classical monetary model is explained.
- Classical monetary model has a lot of problems on consistency from data because of perfect competition and flexible price.
- However, New Keynesian model grounds on classical monetary model and understanding classical model helps us to understand New Keynesian model which is more complicated.
- Classical monetary model has something in common in microfoundation and focusing on dynamics.

3.1 Households

- Representative households' maximization problem is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (3.1)$$

- Two-period model assumes that households dissapear in period 3.
- Now we abandon that and assume that households live infinitely.
- While this assumption is unrealistic, it is not so unrealistic if we imagine that parents consider childs' utility, childs consider grandchilds' utility and ancestors consider descendants' utility.

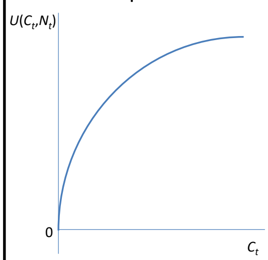
- We assume some charactors on utility function as follows:

$$\frac{\partial U(C_t, N_t)}{\partial C_t} > 0 \; ; \; \frac{\partial^2 U(C_t, N_t)}{\partial C_t^2} \leq 0$$

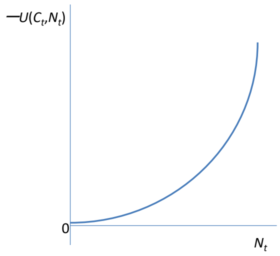
$$\frac{\partial U(C_t, N_t)}{\partial N_t} < 0 \; ; \; \frac{\partial^2 U(C_t, N_t)}{\partial N_t^2} \leq 0$$

- Utility is an increasing function of consumption and marginal utility is diminished.
- Utility is an decreasing function of labor and marginal disutility is increased gradually.

Consumption and Utility



Labor and Disutility



- Households' budget constraint is given by:

$$P_t C_t + Q_{t,t+1} \mathrm{E}_t \left(B_{t+1} \right) \le B_t + W_t N_t - TR_t \quad (3.2)$$

where P_t is the price, i_t is the nominal interest rate, B_t is the discount bond matured in period t, W_t is the nominal wage (rate), TR_t is the lump-sum transfer and $Q_{t,t+1}$ is the stochastic discount factor.
- TR_t can be interpreted variously, e.g., lump-sum tax paid for government, lump-sum dividend from firms.
- Now, government does not exist obviously and it is plausible that TR_t is lump-sum dividend from firms (although there is no excess profit because of perfect competition).

- By iterating Eq.(3.2) forward, we get:

$$\begin{aligned} \mathrm{E}_0 \left(B_k Q_{0,k} \right) = B_0 &+ \left(W_0 N_0 - TR_0 - P_0 C_0 \right) \\ &+ \mathrm{E}_0 \left[Q_{0,1} \left(W_1 N_1 - TR_1 - P_1 C_1 \right) \right] \\ &+ \mathrm{E}_0 \left[Q_{0,2} \left(W_2 N_2 - TR_2 - P_2 C_2 \right) \right] \\ &+ \dots \\ &+ \mathrm{E}_0 \left[Q_{0,k-1} \left(W_{k-1} N_{k-1} - TR_{k-1} - P_{k-1} C_{k-1} \right) \right] \end{aligned}$$

- We assume $B_0 = 0$ which means that there are no assest when the economy is born (similar to two-period model).
- In that case, we have:

$$\mathrm{E}_0 \left(B_k Q_{0,k} \right) = \mathrm{E}_0 \left[\sum_{t=0}^{k} Q_{0,t} \left(W_t N_t - TR_t - P_t C_t \right) \right]$$

- By taking the limit of the LHS in this equality, we get:

$$\lim_{k \to \infty} \mathrm{E}_0 \left(B_k Q_{0,k} \right) = 0 \quad (3.3)$$

- Eq.(3.3) is called transversality condition which implies that households do not leave assets when the economy comes to an end.
- Or, because $Q_{0,k}$ is close to zero, it can be said that Eq.(3.3) is applied.
- Eq.(3.3) corresponds to $B_3 = 0$ in Two-period model where households eat also all of capital before the economy comes to an end.

- Initial condition $B_0 = 0$ and transversality condition Eq.(3.3) implies (somewhat sloppy):

$$\mathrm{E}_0 \left(\sum_{t=0}^{k} Q_{0,t} W_t N_t \right) = \mathrm{E}_0 \left[\sum_{t=0}^{k} Q_{0,t} \left(TR_t + P_t C_t \right) \right]$$

- In this equality, the LHS is revenue and the RHS is expenditure. Thus, all of revenue is expended until period k when the economy comes to an end.
- This implies that infinitely-lived model is same as two-period model in essential.

- Househholds maximize Eq.(3.1) subject to Eq.(3.2).
- As long as initial condition $B_0 = 0$ and transversality condition Eq.(3.3), this problem is solved by (Dynamic) Lagrange's method of undetremined multipliers.
- Lagrangean is given by:

$$L = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U \left(C_t, N_t \right) + \beta^t \lambda_t \left[B_t + W_t N_t - TR_t - P_t C_t - Q_{t,t+1} B_{t+1} \right] \right\}$$

where λ_t is Lagrange multiplyer.

- This can be rewritten as:

$$L = \mathrm{E}_0\{U(C_0,N_0) + \beta U(C_1,N_1) + \ldots + \beta^t U(C_t,N_t) + \beta^{t+1} U(C_{t+1},N_{t+1})$$
$$+\ldots$$
$$+\lambda_0 (B_0 + W_0 N_0 - TR_0 - P_0 C_0 - Q_{0,1} B_1)$$
$$+\beta\lambda_1 [B_1 + W_1 N_1 - TR_1 - P_1 C_1 - Q_{1,2} B_2]$$
$$+\ldots$$
$$+\beta^t \lambda_t [B_t + W_t N_t - TR_t - P_t C_t - Q_{t,t+1} B_{t+1}]$$
$$+\beta^{t+1} \lambda_{t+1} [B_{t+1} + W_{t+1} N_{t+1} - TR_{t+1} - P_{t+1} C_{t+1} - Q_{t+1,t+2} B_{t+2}]$$
$$+\ldots\}$$

- In any 2 periods, households' trade-offs are as follows:

1. Consumption in period *t* and it in period *t*+1 (Or, Consumption and saving in period *t*).
2. Consumption and labor in period *t*.

- That is, consmption in period t+1 shall be diminished if it in period t is increased.
- Further, labor generates disutility although it is essential to obtain income.

- Thus, the trade-offs imply that households have to solve their problem by choosing C_t, C_{t+1}, B_{t+1} and N_t.
- The FONCs for this problem are given by:

$$\frac{\partial L}{\partial C_t} = 0\ ;\ \frac{\partial L}{\partial C_{t+1}} = 0\ ;\ \frac{\partial L}{\partial N_t} = 0\ ;\ \frac{\partial L}{\partial B_{t+1}} = 0$$

- Here, we assume $\frac{\partial L}{\partial C_t}\frac{\partial C_t}{\partial L_t} \doteq 0$
- That is, there is no multiplyer effect (synerism) between consumption and labor. This assumption si called Additively Separable.

- Thus, we get as follows:

$$\lambda_t = \frac{U_{C_t}}{P_t} \quad (3.4)$$

$$\lambda_{t+1} = \frac{U_{C_{t+1}}}{P_{t+1}} \quad (3.5)$$

$$\lambda_t = -\frac{U_{N_t}}{W_t} \quad (3.6)$$

$$\lambda_t = \frac{\beta}{Q_{t,t+1}}\lambda_{t+1} \quad (3.7)$$

- By regarding $1/P_t$ as the value of money and paying attention to Eq.(3.4), we can understand that Lagrange multiplier λt is marginal utility of consumption measured by the value of money.
- Plugging Eqs.(3.4) and (3.5) into Eq.(3.7) yields:

$$\mathrm{E}_t\left(\frac{\beta}{Q_{t,t+1}}\frac{U_{C_{t+1}}}{U_{C_t}}\frac{P_t}{P_{t+1}}\right) = 1 \quad (3.8)$$

- Eq.(3.8) is so called Euler equation.

- We can understand that if the subjective discount factor β equals to the stochastic discount factor $Q_{t,t+1}$, the marginal utility of consumption measured by the value of money is constant overtime.
- Under two-period model we have learned, the marginal utility of consumption is constant overtime.

- Because the stochastic discount factor $Q_{t,t+1}$ corresponds to discount rate for the discount bond purchased by households, the following is applied:

$$Q_{t,t+1} = (1+i_t)^{-1}$$

where i_t denotes the nominal interest rate.
- Plugging this into Eq.(3.8) yields:

$$\beta \mathrm{E}_t \left(\frac{U_{C_{t+1}}}{U_{C_t}} \frac{P_t}{P_{t+1}} \right) = \frac{1}{1+i_t} \qquad (3.10)$$

- Eq.(3.10) is so called intertemporal optimality condition.

- Now, we consider the relationship between households' optimality condition under two-period model and it under this model.
- By arranging Eq.(3.10) yields:

$$U_{C_t} = \frac{1+i_t}{\mathrm{E}_t(\Pi_{t+1})} \frac{1}{1+\delta_t} \mathrm{E}_t\left(U_{C_{t+1}}\right)$$

where δ denotes the rate of time preference which suffices $(1+\delta)^{-1} = \beta$ and $\Pi_t \equiv P_t / P_{t-1}$ denotes the (gross) inflation.

- By taking the logarithsm $(1+i_t)/\mathrm{E}_t(\Pi_{t+1})$, we get:

$$\begin{aligned} \ln \frac{1+i_t}{\mathrm{E}_t(\Pi_{t+1})} &= \ln(1+i_t) - \ln \mathrm{E}_t(\Pi_{t+1}) \\ &\simeq i_t - \mathrm{E}_t(\pi_{t+1}) \\ &= r_t \end{aligned}$$

- Thus, $(1+i_t)/\mathrm{E}_t(\Pi_{t+1})$ can be regarded as the gross real interest rate $1+r_t$ where r_t denotes the net real interest rate.

- Now, the following is applied:

$$U_{C_t} = \frac{1+r_t}{1+\delta} \mathrm{E}_t\left(U_{C_{t+1}}\right)$$

- We assume that the net real interest rate equals to the rate of time preference overtime when we learn two-period model. In this case,

$$U_{C_t} = \mathrm{E}_t\left(U_{C_{t+1}}\right)$$

is applicable.

- That is, the marginal utility of consumption is constant overtime. This result is same as it under two-period model.
- Thus, infinitely-lived model is essentially same as two-period model.
- In addition, the model implies that an increase in the real interest rate relatively decreases the consumption in period t through relative decrease in the marginal utility of consumption, and vice versa.

- That is, as mentioned, business cycle can be controled by monetary policy which adpts interest rate as policy instrument.

- By combining Eq.(3.4) and Eq.(3.6), we have intratemporal optimality condition as follows:

$$-\frac{U_{N_t}}{U_{C_t}}=\frac{W_t}{P_t} \quad (3.11)$$

- Eq.(3.11) shows that marginal rate of substitution on utility is a function of real wage. By paying attention to U_{Nt}<0, an increase in the real wage increases labor input because marginal disutility of labor exceeds marginal utility of consumption, and vice varca.

- If we assume both perfect competition and monotonic increase cost function, $P_t = W_t$ is applied and marginal disutility of labor equals to marginal utility of consumption.

- For, simplicity without loss of generality, we assume:

$$U(C_t, N_t) \equiv \ln C_t - \frac{1}{2}N_t^2$$

- In that case, Eqs.(3.10) and (3.11) can be rewritten as:

$$\beta E_t\left(\frac{P_t C_t}{P_{t+1}C_{t+1}}\right)=\frac{1}{1+i_t} \quad (3.12)$$

$$N_t C_t = \frac{W_t}{P_t} \quad (3.13)$$

3.2 Firms

- Firms' technology is given by following production function:

$$Y_t = A_t N_t^{1-\alpha} \quad (3.14)$$

where Y_t denotes output and A_t denote productivity.

- Firms active in perfect competitive market and maximize their profit shown as following through controling labor input:

$$P_t Y_t - \Psi(Y_t)$$

where $\Psi(Y_t)$ denotes cost function.

- We assume $\Psi'(Y_t) > 0$, that is, the higher the output, the higher the cost, and vice verca.

- Now, we Consider Eq.(3.14).
- Suppose that there are capital and labor as factor of production and constant returns on scale. Then, we have production function as follows:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

where K_t denotes capital.

- Under this production function, α and $1-\alpha$ can be regarded as capital's share of income and labor's share of income, respectively.

- Eq.(3.14) shows that just labor is available as factor of production. (There is no capital in this model because of lack of investment).
- Under Eq.(3.14), when $0<\alpha<1$ is applied, multiplying factor of production *x* times makes output increasing less than *x* times. e.g., when $\alpha=0.5$, doubling factor of production makes output approximately 1.4.
- This is gradually decreasing return on scale.

- When $\alpha<0$, multiplying factor of productivity by *x* times makes output larger than *x* times. e.g., when $\alpha=-1$, multiplying multiplying factor of productivity by 2 times makes output 4 times. This is gradually increasing on scale.
- When $\alpha=0$ multiplying factor of productivity by *x* times makes output *x* times. This is constant returns on scale. e.g., when $\alpha=2$, multiplying factor of productivity by 2 times makes output 4 times.

- Thus, α is a parameter on economies of scale and decides whether returns are gradually decreasing, gradually increasing or constant.

- The FONC is given by:

$$\frac{\partial\left[P_t A_t N_t^{1-\alpha}-\Psi\left(A_t N_t^{1-\alpha}\right)\right]}{\partial N_t}=P_t A_t N_t^{1-\alpha}-\Psi'(\cdot)A_t N_t^{1-\alpha}$$
$$=0$$

- Then, we have:

$$P_t=MC_t^n$$

- Here, $MC_t^n\equiv\Psi'(\cdot)$ denotes the nominal marginal cost. Because a first o-der differential of cost function is the amount of increase in cost when the output increase additionally as much as one unit, this is exactly (nominal) marginal cost.
- The FONC implies that firms choose their price which equals to the nominal marginal cost.

- Dividing both sides on this equality by P_t yields:

$$MC_t=1$$

  where $MC_t\equiv MC_t^n/P_t$ denotes the real marginal cost. Because of flexible price, the real marginal cost is constant.
- This shall exclude when prices are sticky. Changes in the real marginal cost deviates latent GDP (natural rate of output) from actual GDP. This deviation is factor of production which is not utilized efficiently.

- Roughly speaking, GDP gap reflects unemployment or idle equipment.
- In classical monetary model, monetary policy plays a role to decide prices.
- If there is sticky price, monetary policy plays a role not only to decide prices but also to stabilize GDP gap or slush it through stabilizing inflation.

- Now, we think on substance of the (nominal) marginal cost.
- The (nominal) marginal cost is the amount of increase in (nominal) cost when output increases as much as one unit.
- Here, just labor is available as factor of production and the cost as W_t arises when one unit of labor is inputted.
- Thus, the nominal marginal cost is given by:

- Thus, the nominal marginal cost is given by:

$$MC_t^n = \frac{W_t \partial N_t}{\partial Y_t}$$

- Because W_t is the nominal wage—here, it is the unit cost– and ∂N_t is the amount of increase in labor, the numerator is the amount of increase in the cost. ∂Y_t , namely, the denominator is the amount of increase in labor. Thus, the RHS is the amount of increase in the cost when the output increase in one unit.

- This equality can be rewritten as:

$$MC_t^n = \frac{W_t}{\partial Y_t / \partial N_t}$$

- The numerator of the RHS in this equality is unit cost while the denominator is the amount of increase in output when one unit of labor is added additionally, namely, marginal product of labor.

- Now, we derive marginal product of labor $\partial Y_t/\partial N_t$. Production function Eq.(3.14)'s first order difference is marginal product of labor as follows:

$$\frac{\partial Y_t}{\partial N_t} = A_t(1-\alpha)N_t^{-\alpha}$$

- Then, the nominal marginal cost is given by:

$$MC_t^n = \frac{W_t}{A_t(1-\alpha)N_t^{-\alpha}}$$

- This equality implies that an increase in the nominal wage W_t and employment N_t increases the nominal marginal cost while an increase in the productivity A_t decreases the nominal marginal cost.

- Dividing both sides in this equality by P_t yields:

$$MC_t = \frac{W_t}{P_t}\frac{N_t^{\alpha}}{A_t(1-\alpha)}$$

- This equality implies that an includes in the real wage induces to increase the real marginal cost.

- As mentioned, in classical monetary model where is flexible price, the marginal cost is constant overtime and MC_t=1 is applied:
- Plugging MC_t=1 into the previous equation, we have:

$$\frac{W_t}{P_t} = (1-\alpha)A_t N_t^{-\alpha} \qquad (3.15)$$

where W_t/P_t is the real wage.

- Eq.(3.15) is firms' optimality condition and firms choose N_t to suffice Eq.(3.15).
- Eq.(3.15) is also labor supply curve.

Digression: Distribution of Income

- Plugging Eq.(3.15) into Eq.(3.14) yields:

$$Y_t = \frac{1}{1-\alpha}\frac{W_t}{P_t}N_t,$$

which implies that the income is distributed for households depending on their hours of labor.

- As long as constant returns on scale is applicable,

$$Y_t = \frac{W_t}{P_t}N_t \ ,$$

Is applied.

- The previous expression shows that the income definitely corresponds to compensation of employees.

3.3 Equilibrium

- For simplicity, investment, government expenditure and net export are omitted in this model.
- Thus, goods market clearing condition is given by:

$$Y_t = C_t \qquad (3.16)$$

4 Steady State

- Steady state is a state where variables are constant overtime.
- Here, we consider the steady state where prices are constant.
- That is, our steady state is a state where $P_t = 1$ is applied.
- Further, we assume that the productivity is constant and the stochastic discount factor equals to the subjective discount factor in the steady state.
- In this case, $A_t = 1$、$\beta = Q_{t,t+1}$ are applied.

- Arranging Eq.(3.13) yields:

$$C_t = C_{t+1} = C \quad (3.17)$$

which shows that the consumption is constant overtime in the steady state.

- Plugging Eq.(3.17) into Eq.(3.16) yields:

$$Y = C \quad (3.18)$$

which shows that the output is also constant overtime.

- Plugging Eq.(3.18) into Eq.(3.14) yields:

$$Y = N^{1-\alpha}$$

which shows that the labor input is constant in the steady state.

- In the steady state, Eq.(3.15) is arranged by:

$$W = (1-\alpha)N^{-\alpha}P \quad (3.19)$$

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5 Log-linearization

- Here, we log-linearize the model.
- We approximate the model around the steady state. Then, all of variables are converted to percentage deviation from their steady state value.
- To log-linearize, we have to:

1. Totally differentiate relational expressions
2. Divide both sides of differentiated equalities by steady state value.
3. Plugging steady state value into these divided equalities.

- Log-linearization is essentially same as taking logarithm of relational expressions.

- Eq.(3.12) ca be rewritten as:

$$C_t = \mathrm{E}_t(C_{t+1})\frac{1+i}{1+i_t}\mathrm{E}_t(\Pi_{t+1})$$

- Totally differentiating previous equality and plugging steady state value into that yields:

$$dC_t = dC_{t+1} - C\frac{d(1+i_t)}{1+i} + Cd\Pi_{t+1}$$

- Dividing both sides of this by C yields:

$$\frac{dC_t}{C}=\frac{dC_{t+1}}{C}-\frac{d(1+i_t)}{1+i}+\frac{d\Pi_{t+1}}{\Pi}$$

- Paying attention to $\Pi=1$ we can understand that all variables are converted to percentage deviation from their steady state value.

- By defining $v_t=dV_t/V$, the previous can be rewritten as:

$$c_t=E_t(c_{t+1})-\hat{i}_t+E_t(\pi_{t+1}) \qquad (3.20)$$

where v_t denotes percentage deviation of arbitrary variable V_t from its steady state value V.

- Now, $\hat{i}_t$ is different from the nominal interest rate i_t and is percentage deviation of gross nominal interest rate from its steady state value.
- By log-linearizing Eq.(3.18), we have $y_t=c_t$. Thus, Eq.(3.19) can be rewritten as:

$$y_t=E_t(y_{t+1})-\hat{i}_t+E_t(\pi_{t+1}) \qquad (3.21)$$

- By log-linearizing Eqs.(3.14) and (3.15), we get:

$$y_t=a_t+(1-\alpha)n_t \qquad (3.22)$$

$$w_t-p_t=a_t-\alpha n_t \qquad (3.23)$$

- Eq.(3.13) can be log-linearized as:

$$w_t-p_t=n_t+y_t \qquad (3.24)$$

where we use $y_t=c_t$.

- Eqs.(3.23) and (3.24) are log-liearized labor supply and labor demand curves, respectively.

Appendix Derivation of Eq.(3.15)

- Eq.(3.15) can be derived easily by assuming perfect competition.
- Firms maximize thei profit as follows:

$$P_tY_t-W_tN_t$$

- The FONC is given by:

$$\frac{\partial\left(P_tA_tN_t^{1-\alpha}-W_tN_t\right)}{\partial N_t}=0$$

- Thus, we have firms' optimality condition as follwos:

$$\frac{W_t}{P_t}=(1-\alpha)A_tN_t^{-\alpha}$$

which is Eq.(3.15) itself.